COSC 341 – Tutorial 4, Solutions

1. Give a recursive definition of the set B of unlabelled complete binary trees.

   Basis $() \in B$

   Recursive step If $b \in B$, then $(bb) \in B$

2. Show that the power set $\mathcal{P}(\mathbb{N})$ of $\mathbb{N}$ is uncountable.

   As with all diagonal arguments, we argue by contradiction. Suppose, contrary to what we want to prove, that $\mathcal{P}(\mathbb{N})$ were countable. In that case we could list all subsets of $\mathbb{N}$ as:

   $$A_0, A_1, A_4, \ldots$$

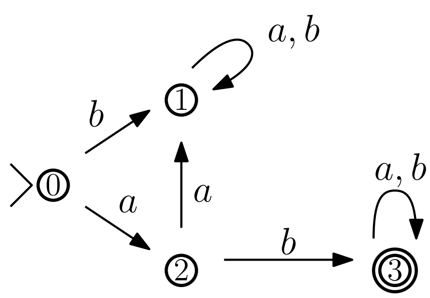
   Now we define a set $A = \{i | i \in \mathbb{N}, i \notin A_i\} \subseteq \mathbb{N}$. There is no k for which $A = A_k$. If there was such a k, it would either be

   $$k \in A \Rightarrow k \notin A_k = A \text{ (by the definition of } A) \Rightarrow \text{ contradiction}$$
   $$\text{or } k \notin A \Rightarrow k \in A = A_k \text{ (by the definition of } A) \Rightarrow \text{ contradiction}$$

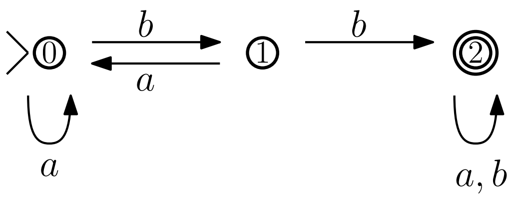
   So our list of subsets was *not* complete as we claimed it was, and hence no such list can exist.

3. Design a finite automaton on the alphabet $\{a, b\}$ that accepts:

   (a) all words starting with ab



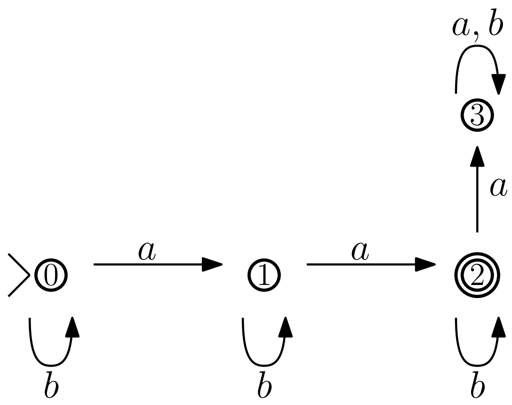
   (b) all words containing the substring bb



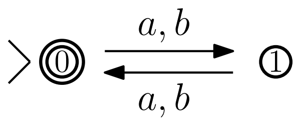
Homework

1. Design a finite automaton on the alphabet $\{a, b\}$ that accepts:

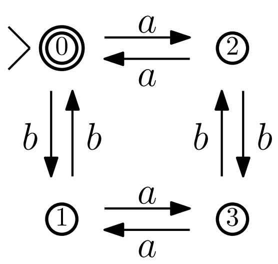
   (a) all words containing exactly two a's



   (b) all words of even length



   (c) all words consisting of an even number of a's and an even number of b's



2. Give a simple recursive definition of the language Eq consisting of strings over $\{a, b\}$ which have an equal number of a's and b's.

   Basis The empty string belongs to Eq

   Recursive step If $u \in$ Eq and u can be written xy, then $axby$ and $bxay$ belong to Eq