COSC 341 – Tutorial 1, Solutions

1. Let $A = \{0, 1, b, f, \text{SpongeBob}\}$ and $B = \{1, \text{Patrick}, \text{SpongeBob}, 2, f, m\}$. List the elements of:

   (a) $A \cup B$ (the union of X and B)

   $$A \cup B = \{0, 1, 2, b, f, m, \text{SpongeBob}, \text{Patrick}\}.$$

   (b) $A \cap B$ (the intersection of A and B)

   $$A \cap B = \{1, f, \text{SpongeBob}\}.$$

   (c) $A \setminus B$ (the complement of B relative to A)

   $$A \setminus B = \{0, b\}$$

   (d) $B \setminus A$ (the complement of A relative to B)

   $$B \setminus A = \{\text{Patrick}, 2, m\}.$$

2. Set builder notation

   (a) Give the set $\{0, 2, 4, 6, 8, \dots\}$ in set builder notation

   $$\{0, 2, 4, 6, 8, \dots\} = \{2n | n \in \mathbb{N}\}$$

   (b) List the elements of $\{x | x \leq 5, x \in \mathbb{N}\}$

   $$\{x | x \leq 5, x \in \mathbb{N}\} = \{0, 1, 2, 3, 4, 5\}$$

3. Let $A = \{\text{Connor}, \text{Tauiri}, \text{Hans-Christian}\}$ and $B = \{\text{SpongeBob}, \text{Patrick}\}$.

   (a) List all elements of $\mathcal{P}(A)$ (the power set of A)

   $$\mathcal{P}(X) = \left\{ \begin{array}{c} \emptyset, \{\text{Connor}\}, \{\text{Tauiri}\}, \{\text{Hans-Christian}\} \\ \{\text{Connor}, \text{Tauiri}\}, \{\text{Connor}, \text{Hans-Christian}\}, \\ \{\text{Tauiri}, \text{Hans-Christian}\}, \{\text{Connor}, \text{Tauiri}, \text{Hans-Christian}\} \end{array} \right\}$$

   (b) List all the members of $A \times B$.

   $$A \times B = \left\{ \begin{array}{cc} (\text{Connor}, \text{SpongeBob}), & (\text{Connor}, \text{Patrick}) \\ (\text{Tauiri}, \text{SpongeBob}), & (\text{Tauiri}, \text{Patrick}) \\ (\text{Hans-Christian}, \text{SpongeBob}), & (\text{Hans-Christian}, \text{Patrick}) \end{array} \right\}$$

   (c) List all functions from B to A.

$\{(\text{SpongeBob}, \text{Connor}), (\text{Patrick}, \text{Connor})\}$
$\{(\text{SpongeBob}, \text{Connor}), (\text{Patrick}, \text{Tauiri})\}$
$\{(\text{SpongeBob}, \text{Connor}), (\text{Patrick}, \text{Hans-Christian})\}$
$\{(\text{SpongeBob}, \text{Tauiri}), (\text{Patrick}, \text{Connor})\}$
$\{(\text{SpongeBob}, \text{Tauiri}), (\text{Patrick}, \text{Tauiri})\}$
$\{(\text{SpongeBob}, \text{Tauiri}), (\text{Patrick}, \text{Hans-Christian})\}$
$\{(\text{SpongeBob}, \text{Hans-Christian}), (\text{Patrick}, \text{Connor})\}$
$\{(\text{SpongeBob}, \text{Hans-Christian}), (\text{Patrick}, \text{Tauiri})\}$
$\{(\text{SpongeBob}, \text{Hans-Christian}), (\text{Patrick}, \text{Hans-Christian})\}$
$\{(\text{SpongeBob}, \text{Connor})\}$
$\{(\text{SpongeBob}, \text{Tauiri})\}$
$\{(\text{SpongeBob}, \text{Hans-Christian})\}$
$\{(\text{Patrick}, \text{Tauiri})\}$
$\{(\text{Patrick}, \text{Hans-Christian})\}$
$\{(\text{Patrick}, \text{Connor})\}$
$\emptyset$

4. Are the following functions $f : \mathbb{N} \to \mathbb{N}$ surjective, injective, bijective?

   (a) $f(x) = 2x + 1$

injectivity:

Let $f(x), f(y) \in \mathbb{N}$ with $f(x) = f(y)$.
$\Rightarrow\ 2x - 1 = 2y - 1$
$\Rightarrow\ 2x = 2y$
$\Rightarrow\ x = y$
$\Rightarrow\ f$ is injective

surjectivity:

For $0 \in \mathbb{N}$ there is no $x \in \mathbb{N}$
with $f(x) = 0$
$\Rightarrow\ f$ is not surjective

$\Rightarrow f$ is not bijective

(b) $f(x) = \frac{x}{2}$ (integer division, e.g. $\frac{3}{2} = 1$)

injectivity:

For $1 \in \mathbb{N}$ it holds

$f(3) = \dfrac{3}{2} = 1 = \dfrac{2}{2} = f(2)$, but

$3 \neq 2$

$\Rightarrow$ f is not injective

surjectivity:

Let $y \in \mathbb{N}$ be an arbitrary natural number.

Let x be defined as $x = 2y$

$\Rightarrow x \in \mathbb{N}$ and $f(x) = \dfrac{2y}{2} = y$

$\Rightarrow$ f is surjective

$\Rightarrow f$ is not bijective

(c) $f(x) = 1$ (constant)

injectivity:

For $0 \in \mathbb{N}$ and $1 \in \mathbb{N}$ it holds

$f(0) = 1 = f(1)$ but

$0 \neq 1$

$\Rightarrow$ f is not injective

surjectivity:

For $0 \in \mathbb{N}$ there is no $x \in \mathbb{N}$

with $f(x) = 0$

$\Rightarrow$ f is not surjective

$\Rightarrow f$ is not bijective

5. Give examples of functions $f : \mathbb{N} \rightarrow \mathbb{N}$ that are bijective.

$$f(x) = x \text{ (identity)}$$

$$f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$$

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \\ x & \text{,else} \end{cases}$$